Israeli Mathematical Olympiad – 8th-9th Grades
Final Stage, Year 5783

1. Mark 8 points on the plane so that for each point there are four other points whose distances from it are 2, 3, 4, and 5.

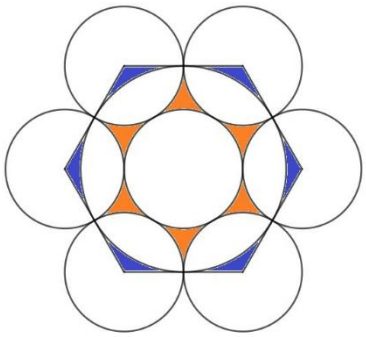
2. There are 100 kiosks arranged in a circle. Each kiosk belongs to a different merchant, and each merchant has some number of coins. Each merchant also has some constant price at which they sell their fish, and an infinite supply of fish. On the first day, one of the merchants built a golden fishing rod. Each day thereafter, the merchant owning the golden fishing rod walked over to the kiosk belonging to the next merchant, gave them the rod, and bought as many fish as they possibly could with their money. If on some day, a merchant does not succeed in buying any fish, we call it a "sad day".

Prove that if day 2022 is sad, then day 2023 is also sad.

3. What is the smallest prime $p > 10$ such that

$$|105 - 2p|$$

is composite?



4. In the following figure there are 7 circles of equal size that are tangent to each other. The figure also shows a hexagon formed by connecting the centers of 6 of those circles, and a circle inscribed in said hexagon.

Prove that the blue area is equal to the orange area.

5. A community centre hosts N classes, with several students participating in each class. It is known that no two students participate in precisely the same set of classes, and that each student participates in at least one class. Two students are friends if they have at least one class in common. It turns out that if student A is a friend of student B, and student B is a friend of student C, then student A is a friend of student C.

As a function of N, what is the biggest possible number of students that might be studying at the community centre?

6. Find all solutions in positive numbers for the following set of equations:

$$\begin{cases} x^2 + xy = z \\ y^2 + yz = x \\ z^2 + zx = y \end{cases}$$

7. In an equilateral triangle ABC the point D is chosen on the segment AB such that $AD = \sqrt{3}BD$. The point E is chosen on the segment AC such that $\sqrt{3}CE = 2AE$. The line DE intersects the continuation of the side BC at F.

Find the value of the angle $\angle DFB$, in degrees.

בהצלחה!