The national math Olympiad - grades 8-9
Stage B, 2022

1. The integers from 1 to 100 are written on the squares of a 10×10 board. On each row, the numbers from left to right are in increasing order, and on each file, the numbers from the bottom up are in increasing order. The files are numbered 1 to 10 from left to right and the rows are numbered 1 to 10 from the bottom up. Find the minimum possible sum of the row and file numbers of the square containing 39.

2. Let there be integers a, b, c, d such that:

$$a + b = bc = d - c = \frac{d}{a}, c \neq 0, a \neq 0$$

Find $a + b + c + d$.

3. In a pentagon $ABCDE$ the angels $\angle A, \angle D, \angle E$ are right, and $AB = 50, BC = 34, CD = 64$. It is also known that the pentagon is **circumscribed**, what is the radius of the inscribed circle?

4. The following expression is equal to an integer. Find said integer:

$$\frac{\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \dfrac{1}{\sqrt{5}+\sqrt{6}} + \cdots + \dfrac{1}{\sqrt{97}+\sqrt{98}} + \dfrac{1}{\sqrt{99}+\sqrt{100}}}{-\dfrac{1}{\sqrt{51}-\sqrt{1}} + \dfrac{1}{\sqrt{52}-\sqrt{2}} - \dfrac{1}{\sqrt{53}-\sqrt{3}} + \cdots - \dfrac{1}{\sqrt{99}-\sqrt{49}} + \dfrac{1}{\sqrt{100}-\sqrt{50}}}$$
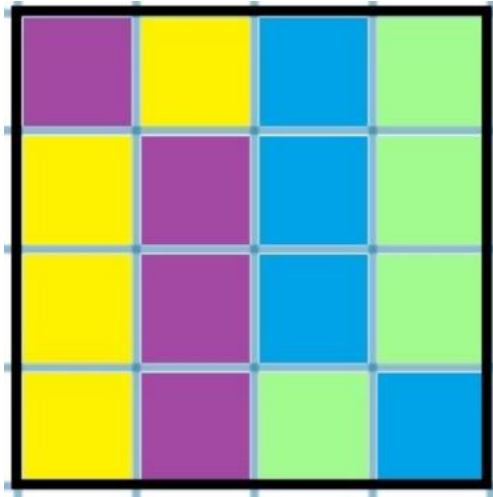
5. a circle inscribes a square. A blue circle is inscribed by the square's diagonals and the circle. An orange circle is inscribed by one of the square's sides and the circle. Find the ratio between the two circles' areas.
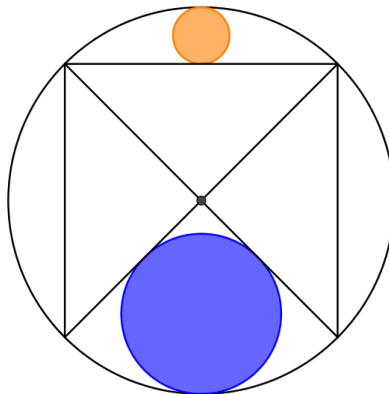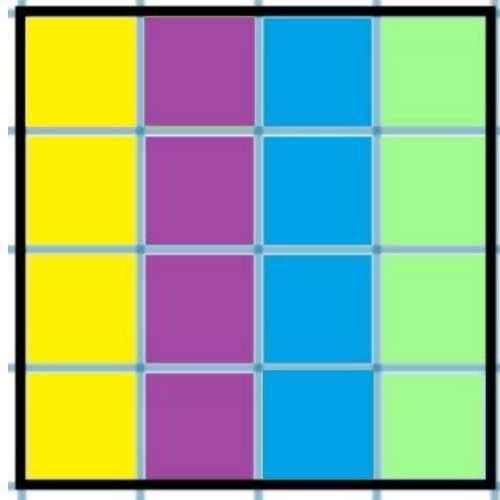

6. Let there be a 4x4 board. Eliora wants to divide it into 4 congruent shapes, each containing 4 full squares. Between any two squares in the same shape, there must be a path that passes through that shape and only moves from a square to a square sharing a side with it. Two shapes are considered congruent if they can be obtained from each other through rotation, mirroring, and translation. Two divisions are considered different even if they can be obtained from each other by mirroring or rotating the board. In how many different ways can Eliora divide the board?

An illegal division

a legal division

7. a regular polygon with 2022 sides is **inscribed** by a circle. A second circle, concentric to the first, is given such that its **area is 2 times smaller** than the big circle's. All diagonals of the polygon that are not tangent to the inside circle are drawn. Find the difference between the number of diagonals that pass through the inside circle to the number of diagonals the do not.

Good luck!