



Israeli Mathematical Olympiad – 6th-7th Grades  
Final Stage, Year 5783

1. Asaf, Bar, Carmel, and Dror, are siblings. Each is either a knight or a knave.

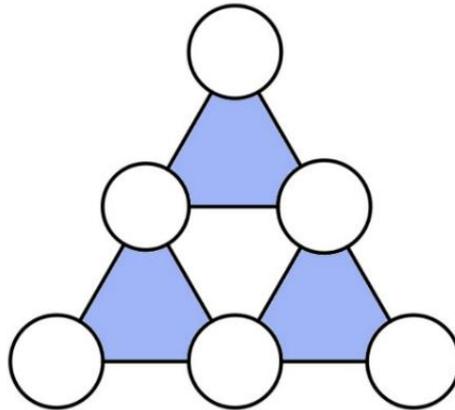
Knives always lie and knights always say the truth.

One day, Asaf said: "Among the four of us there are at least two knaves."

Then Bar said: "At least three of us are knaves."

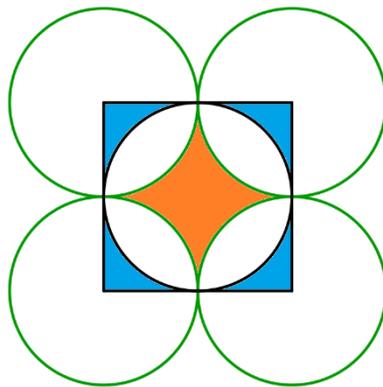
How many of the siblings are knaves?

2. Avihai arranged the numbers 1, 2, 3, 4, 5, 6 in the circles, each number being used exactly once. He then noticed that for each of the blue triangles, the sums of the numbers in their vertices are equal to each other. Find all possible values for that sum. For each option, give an example for such an arrangement, and prove that there are no other options.



3. Mark 6 points on the plane so that any three of them form an isosceles triangle.

4. In the following figure there are four circles of equal size that are tangent to each other. The centers of the circles form a square. Which area is larger: The blue area or the orange area?

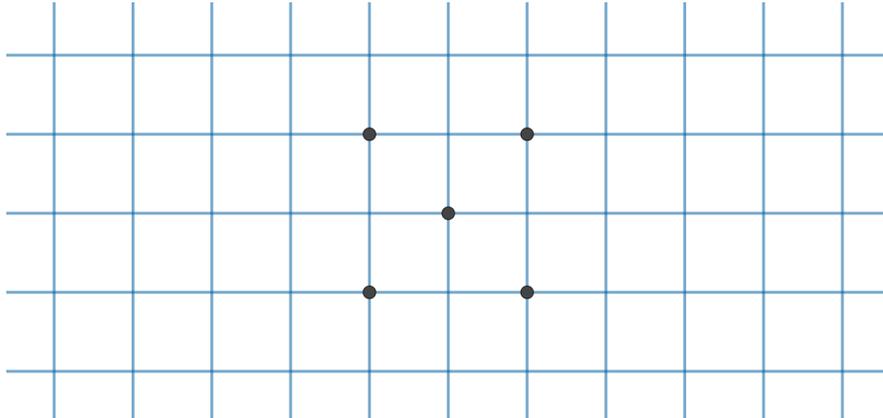


More questions on the  
second page!

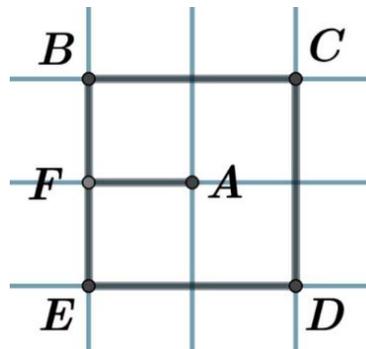


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5. Draw a polygon with minimal perimeter, such that its edges coincide with the lines of the grid, passing through all five of the points marked below. Prove that there is no such polygon with a smaller perimeter.



Note: The polygon may not intersect with itself. For example, the path AFBCDEFA is not a polygon. That is, the following drawing is not allowed:



6. A warehouse contains five flour sacks, weighing 1, 2, 3, 4, 4 Kilograms. Each sack has its weight written on it. Ittay chose two of the sacks and moved some flour from one to the other. Show how using three weightings on a balance scale, it is possible to find out from which bag Ittay removed flour, and to which bag it was added?

Note: It is possible that the amount of flour Ittay moved is not a whole number of Kilograms.

**בהצלחה!**