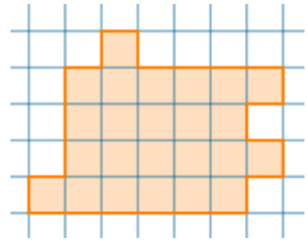
<u>The Israeli Mathematics Olympiad for 9th Grade</u>

Final Round, 2021

You are required to prove every statement and explain every answer (wherever it is relevant)

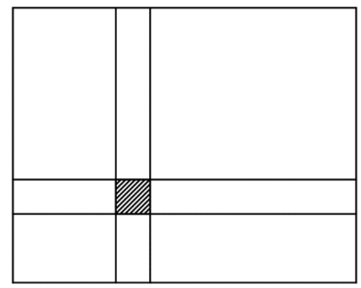
1. Divide this shape into 4 congruent parts:



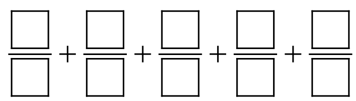
2. Prove that the following equation has infinitely many solutions in positive integers:

$$x^4 + y^4 + z^4 + t^4 = w^5$$

3. On a 43×47-cell square grid there is a blacked-out cell that doesn't touch the grid's perimeter. Taking the continuations of the sides of the cell until the points of intersection with the perimeter of the grid, we divide the shape into 8 smaller rectangles (as in the figure). Prove that one cannot construct a rectangle from the 8 smaller rectangles.

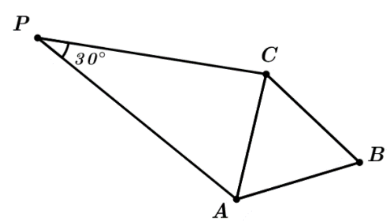


4. Insert the numbers 1-10 into the cells, with each number appearing once, such that the following two conditions are met:



- In each fraction, the numerator and the denominator are coprime, meaning that their greatest common divisor is equal to 1.
- The sum of all the fractions is an integer number.

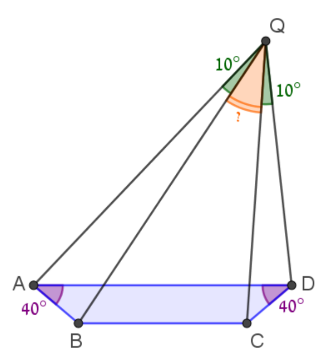
5. The triangle ABC is equilateral. We construct a triangle APC outwardly, such that $\angle APC = 30°$. Prove that one can construct a right-angled triangle from the segments AP, BP and CP.



6. In a magical land there are 4 types of people: *positive* people, *negative* people, *truth tellers*, and *liars*. Positive people always answer "yes", negative people always answer "no", truth tellers always tell the truth and liars always lie. You met four residents of this magical land: one positive, one negative, one truth teller and one liar. You are only allowed to ask yes/no questions. What is the smallest number of such questions required to discover which is which?

7. Prove that for all $a, b > 0$, the inequality $\frac{a}{b} + \frac{b}{a} + (a-1)(b-1) \geq 2$ holds.

8. We are given an isosceles trapezoid $ABCD$ with bases AD and BC, such that the angles near the base AD are $40°$. We are also given a point Q such that the segments QB and QC have different lengths and intersect AD. Assuming $\angle AQB = 10° = \angle CQD$, find $\angle BQC$.



Good luck!

The exams, the solutions and more will appear on our website: http://www.israelmath.com/olymp