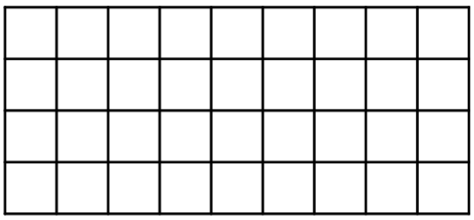
האולימפיאדה הארצית במתמטיקה לביתות ט
שלב ב, שנת תשפ"א

1. A factorial of natural number N is defined to be the product of all natural numbers up to N, including: $N! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot N$. What is the smallest N for which $N!$ Is divisible by 2020^{19}?

(in other words, what is the smallest N for which $\frac{N!}{2020^{19}}$ is an integer number)

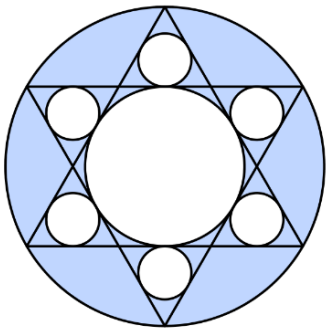
2. How many rectangles with even area are in the following picture?



Note: the areas of the small squares are 1.

3. 15 athletes compete in two races. In each race, every athlete is rated – the first athlete to complete the course is rated first, the second is rated second, and so on until the 15th athlete who's rated 15th. No to athletes finish the course simultaneously. In personal pep talks, that occurred between the two races, each athlete was told that their rating on the second race will be better than their rating on the first. As it turned out, the rating of each athlete changes by at most 3. What is the greatest possible number of athletes for which the thing said in the pep-talk can be true?

4. Vertices of a regular hexagon are marked on a circle. The marked points are connected, such that the lines form two equilateral triangles, intersecting in the shape of a Magen David, as depicted on the following picture. Area of each of the 6 smaller circles is 1. What is the total area that is coloured blue?



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5. For every natural number N, define $S(N)$ to be the sum of its divisors. Moreover, define $S^*(N) = S(N) - N - 1$. Find the smallest natural number k for which there's more than one value of N with $S^*(N) = k$.

Remark. A natural number is a positive integer number.

6. Find the numeric value of the following infinite expression:

$$\sqrt{1+\sqrt{5+\sqrt{11+\sqrt{19+\sqrt{29+\cdots}}}}}$$

Remark: the numbers in the roots are increased by 4,6,8,10,12 and so on.

בהצלחה!