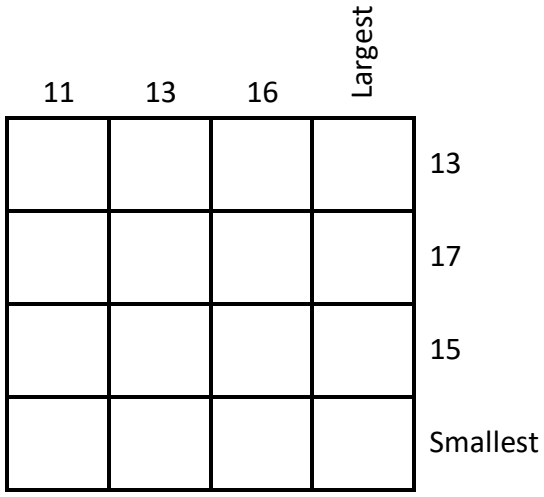
האולימפיאדה הארצית במתמטיקה לביתות ז-ח
שלב ב, שנת תשפ"א

1. In the following picture there is a 4x4 table of numbers.

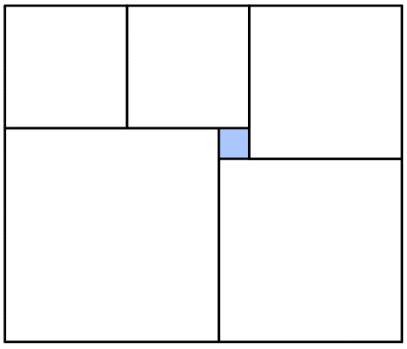
The sums of the numbers in the rows of the table are distinct. The sum of the numbers in each row is written besides it, except for the row with the smallest sum (the bottom row).

The sums of the numbers in the columns are distinct. The sum of the number in each column is written above it, except for the column with the largest sum (the rightmost column).



What is the sum of the numbers in the bottom row?

2. On the following figure is a rectangle divided into 6 squares. The area of the blue square is 1 cm^2. What is the area of the rectangle?



3. Compute the value of the following expression.

$$\frac{1}{\frac{1}{100\cdot 102}+\frac{1}{102\cdot 104}+\frac{1}{104\cdot 106}+\cdots+\frac{1}{198\cdot 200}}$$

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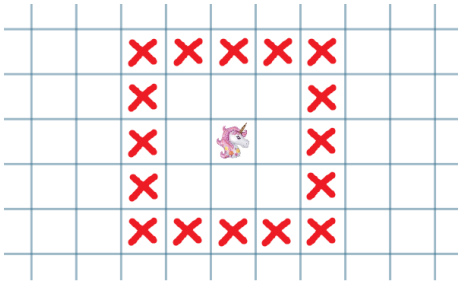
4. For every natural number N, define $S(N)$ to be the sum of its divisors. Moreover, define $S^*(N) = S(N) - N - 1$. Find the smallest natural number k for which there's more than one value of N with $S^*(N) = k$.

Remark. A natural number is a positive integer number.

5. Define a *unicorn* to be a chess piece that moves two squares in one direction (vertically or horizontally) and then 0, 1, or 2 squares in a perpendicular direction. In the following picture we marked all the squares threatened by the unicorn.



What is the largest number of unicorns that can be placed on a 6×6 board without any two threatening each other?

6. 15 athletes compete in two races. In each race, every athlete is rated – the first athlete to complete the course is rated first, the second is rated second, and so on until the 15th athlete who's rated 15th. No to athletes finish the course simultaneously. In personal pep talks, that occurred between the two races, each athlete was told that their rating on the second race will be better than their rating on the first. As it turned out, the rating of each athlete changes by at most 3. What is the greatest possible number of athletes for which the thing said in the pep-talk can be true?

Good Luck!